# Nonuniform Compressive Sensing for Heterogeneous Wireless Sensor Networks

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*Abstract*—In this paper, we consider the problem of using wireless sensor networks (WSNs) to measure the temporal-spatial profile of some physical phenomena. We base our work on two observations. First, most physical phenomena are compressible in some transform domain basis. Second, most WSNs have some form of heterogeneity. Given these two observations, we propose a nonuniform compressive sensing method to improve the performance of WSNs by exploiting both compressibility and heterogeneity. We apply our proposed method to real WSN data sets. We find that our method can provide a more accurate temporal-spatial profile for a given energy budget compared with other sampling methods.

*Index Terms*—Compressive sensing (CS), heterogeneity, nonuniformal sampling, sample schedule, wireless sensor networks.

# I. INTRODUCTION

**I** N THIS paper, we consider the problem of using wireless sensor networks (WSNs) to measure the temporal-spatial profile of some physical phenomena in an energy efficient manner. Much work [7], [9], [21] has been done in improving the efficiency of WSNs in the past decade. The key distinction of this paper is that we exploit compressibility and heterogeneity to derive a nonuniform compressive sensing method to improve the performance of WSNs. More specifically, the nonuniform compressive sensing method that we propose can give a more accurate temporal-spatial profile for a given energy budget compared with other methods.

Our work is based on two hypotheses. Firstly, we assume that most physical phenomena are compressible in some transform domain basis. This is also the assumption behind the recently proposed theory of compressive sensing (CS), which is an efficient signal reconstruction method that can recover a signal from a small number of samples [3], [4]. This is also the assumption behind a number of recent work [1], [5], [14], [18] on using compressive sensing to improve the operations of WSNs. Secondly, we assume that each WSN has some

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form of heterogeneity. As an example, for a rechargeable WSN using solar energy, some nodes have a lower energy harvest rate because of the presence of shadowing; these nodes have a lower energy harvest rate compared to those nodes whose solar panels are not obstructed [12], [23]. In addition, it is possible for nodes in a rechargeable WSN to have different initial amount of energy [10].

Given these two hypotheses, we propose a nonuniform compressive sensing (NCS) method to improve the performance of WSNs by exploiting both compressibility and heterogeneity. We demonstrate how NCS can be exploited using WSN which is powered by solar energy and the dominant form of energy consumption is sensing. Furthermore, we investigate two fundamental questions on the performance of NCS.

- 1) What is the probability of accurate signal recovery with the NCS framework?
- 2) What is the benefit of introducing NCS into WSNs? In particular, we want to know the amount of improvement NCS can make in improving the accuracy of the temporal-spatial profile for a given energy budget.

To answer the first question, we formally derive the probability that NCS can recover a sparse signal exactly. Although our theoretical results show that NCS has a lower recovery probability compare with uniform sampling based compressive sensing, we will show using real WSN data that the reconstruction accuracy of NCS and uniform sampling based compressive sensing are similar.

In order to answer the second question, we evaluate the proposed NCS extensively with a real WSN application dataset, which features resource heterogeneity. We present a distributed implementation of NCS framework that introduces very little communication overheads, and show that, compared to previously proposed approaches based on traditional CS and sparse approximation respectively, NCS achieves similar signal approximation accuracy but with significantly less energy consumption.

The rest of this paper is organized as follows. Section II discusses assumptions (signal compressibility and resource heterogeneity). In Section III, we discuss the basic setup of the problem and the background knowledge of CS, which is followed by the introduction of the notion of NCS and the derivation of the probability of signal accurate recovery by NCS in Section IV. We evaluate and study proposed NCS framework by the dataset from a WSN deployment, which features resource supply heterogeneity in Section V.

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We discuss prior work in Section VI. Finally, Section VII concludes this paper.

## **II. ASSUMPTIONS**

WSNs are deployed to obtain an accurate temporal-spatial profile of some physical phenomena, e.g., temperature, humidity, wind speed, and/or wind direction [15], [23]. In this paper, we make the following two assumptions on the behavior of WSNs for nonuniform Compressive Sensing (NCS) architecture.

Assumption 1: The signals (or physical phenomena) monitored by WSNs are compressible in some transform domains. Common examples of transform domains include Discrete Cosine Transform (DCT), wavelets or Haar wavelet.

Assumption 1 implies that the sampling frequency and spatial distribution of sensors are high enough to capture any temporal and spatial correlation in the usually slowly-varying underlying environmental state of interest [20], and we will formally defined compressible signal in Section III.

Assumption 2: There is heterogeneity, e.g., the energy supply (harvest) rate and/or energy consumption rate, in the WSNs, and we can exploit the heterogeneity to increase the performance (e.g., increase network duty cycles and lifetime) of WSNs.

We will introduce a real application in Section V, where NCS exploits heterogeneity in solar energy harvest rate to improve the performance of WSNs. We believe that these two assumptions are not restrictive at all and can be satisfied by most WSNs.

An important argument that we will make in the following section is that, due to the heterogeneity of the WSNs, the probability distribution for making sampling decision should be nonuniform. Furthermore, we will demonstrate that nonuniform sampling has a better performance compared with uniform sampling.

# III. BASIC SETUP AND BACKGROUND ON COMPRESSIVE SENSING

We point out in the previous section that heterogeneity in resource supply or resource consumption is a common phenomenon in WSNs. Given this heterogeneity in resource supply or consumption in WSNs, we propose that nonuniform sampling can be a tool that we can exploit to improve the performance in such WSNs. In particular, we want to show that, by using nonuniform sampling, we can, for a given energy budget, improve the accuracy of the temporal-spatial profile obtained from WSNs.

In order to realise this goal, we need to develop a model to understand how nonuniform sampling affects the accuracy of the temporal-spatial profile obtained. If we have such a model and if we also know the energy consumption of a particular nonuniform sampling pattern, then we will be able to determine good sampling patterns that give accurate temporalspatial profile for a given energy consumption. Therefore, in Section IV, we will develop a model to show how nonuniform sampling affects the accuracy of the temporal-spatial profile. The model that we will develop uses compressive sensing as the building block. The aim of this section is two-fold. Firstly, we present a basic setup of the problem and review some results in compressive sensing which is necessary for the development in Section IV.

We begin with setting up the problem. Consider a wireless sensor network with N nodes where each node measures a number of physical phenomena, e.g. temperature, humidity, wind speed, wind direction. We will consider one physical phenomenon at a time. Let  $x_{it}$  denote the value of a particular physical phenomenon at sensor i (where i = 1, ..., N) and time t (where t = 1, ..., T). The complete temporal-spatial profile of a physical phenomenon consists of n = NT values of  $x_{it}$ with i = 1, ..., N and t = 1, ..., T. It is obviously good to have the complete temporal-spatial profile since this provides the maximum amount of information. However, this means that all sensor nodes will need to sample at all time and this can result in high sampling or transmission energy consumption. In order to lower the energy consumption, we do not require all the sensors to sample the physical phenomenon at all time. If the value of a physical phenomenon  $x_{j\tau}$  is not measured (or sampled) by sensor j at time  $\tau$ , we will predict the value of  $x_{i\tau}$  from those sensor readings that are available.

In the following discussion, we will collect all the values of  $x_{it}$  into a  $n \times 1$  vector **x** where each element of **x** corresponds to the value of the physical phenomenon at a particular sensor at a particular time. We will assume some of the elements of the vector **x** are known (i.e., an element of **x** is known if the corresponding sensor samples at the corresponding time) and the goal is to predict the unknown elements of  $\mathbf{x}$  from those that are known. A key idea behind the prediction method is to exploit the fact that most physical phenomena are compressible in some transform basis, e.g. Fourier, DCT, wavelets etc. A signal  $\mathbf{x} \in \mathbb{R}^n$  is said to be compressible in a transform basis  $\Psi$ , if the coefficients of the **x** in the basis  $\Phi$  decays according to the power law. In order to define this more precisely, we specify a transform basis  $\Psi$  by a  $n \times n$  matrix  $\Psi$  whose columns are the basis vectors. In this case, the coefficients of **x** in the basis  $\Psi$  is given by the vector **g** where  $\mathbf{x} = \Psi \mathbf{g}$ . Let us rearrange the elements in g in decreasing order of magnitude,  $|g|_{(1)} \ge |g|_{(2)} \ge \dots \ge |g|_{(n)}$ , then **x** is compressible if the following condition holds:

$$|g|_{(k)} \le Ck^{-p} \ \forall k = 1, .., n \tag{1}$$

for some  $p \ge 1$  [4] and some constant *C*. We will use data collected from real wireless sensor networks to show that physical phenomena such as temperature, humidity, wind speed and wind direction are compressible.

We now explain how compressive sensing method, such as the one described in [3], can be used to estimate the unknown elements in **x** from those that are known. Let us assume that *m* elements of **x** are known and the indices of these *m* elements in **x** are  $k_1, k_2, ..., k_m$ . Let  $\Omega$  be the set of the indices of the samples, i.e.,  $\Omega = \{k_1, k_2, ..., k_m\}$ . Let also  $\mathbf{I} \in \mathbb{R}^{n \times n}$  denote the identity matrix. We define  $\mathbf{I}_{\Omega}$  be a *m*-by-*n* matrix such that the *k*-th row of **I** is also a row in  $\mathbf{I}_{\Omega}$  if  $k \in \Omega$ . With this definition, the vector  $\mathbf{I}_{\Omega} \mathbf{x} \in \mathbb{R}^m$  contains the known elements of **x**.

The compressive sensing method in [3] says that one can estimate the unknown elements in x given  $I_{\Omega}x$  (i.e., the

known elements in  $\mathbf{x}$ ) and the fact that  $\mathbf{x}$  is compressible in the transform domain  $\Psi$  by solving the following linear programming problem:

$$\hat{x} = \Psi \hat{y}$$
 where  $\hat{y} = \arg \min_{\mathbf{y} \in \mathbb{R}^n} \|\mathbf{y}\|_1$  s.t.  $\mathbf{I}_{\Omega} \Psi \mathbf{y} = \mathbf{I}_{\Omega} \mathbf{x}$ . (2)

For the case where the vector  $\mathbf{x}$  is sparse (note: a vector x is sparse if its transform in some domain contains a small number, say  $S \ll n$ , of nonzero coefficients), [3] gives some theoretical results on how the probability of recovering the vector  $\mathbf{x}$  successfully depend on m, see [3] for further details. Although the above result is stated for sparse vectors, it has been successfully applied to real-life data (which is generally not sparse, but compressible) such as magnetic resonance imaging [3]. In practice, the above result can be applied to compressible signal if we interpret the number of nonzero coefficients S in the sparse signal as the number of dominant coefficients in the compressible signal.

The compressive sensing method described above assumes that **exactly** m out of n (where m << n) elements of the vector  $\mathbf{x}$  are sampled. The key difficulty of using this method in wireless sensor networks is that a good amount of co-ordination is needed by the nodes to ensure that exactly m elements of  $\mathbf{x}$  are sampled. In the next section, we will introduce two sampling methods such that we do **not** require exactly m elements of  $\mathbf{x}$  are sampled, rather, we require the mean number of samples is m. Such probabilistic methods require a less co-ordination among the nodes and are more suited for distributed implementation. Furthermore, such type of probabilistic sampling methods have not been studied and are a key contribution of this paper.

# IV. NCS WITH SPARSE MATRIX

This section considers the following data recovery problem: given that a number of elements of the vector  $\mathbf{x} \in \mathbb{R}^n$ are known, the goal is to recover the unknown elements of  $\mathbf{x}$  using the knowledge that the vector  $\mathbf{x}$  is sparse in a known transform basis. Our aim is to derive the probability of recovering  $\mathbf{x}$  successfully under two different models of sampling the elements of  $\mathbf{x}$ , namely uniform Bernoulli model and nonuniform Bernoulli model. Note that the analytical result is proved for sparse signal, but in practice the result is applicable to compressible signal as mentioned earlier. We first define the problem more precisely.

# A. Problem Definition

Given an unknown vector  $\mathbf{x}$  with *n* elements, we first sample a number of elements of  $\mathbf{x}$  and then use these sampled elements together with the fact that  $\mathbf{x}$  is sparse in a known basis to cover the unmeasured elements of  $\mathbf{x}$ . In this paper, we will consider two different probability distributions for sampling the elements of  $\mathbf{x}$ .

Let us first formally define the two different sampling distributions. Let  $\delta_k \in \{0, 1\}$  be a random variable that denotes whether the *k*-th element of **x** is sampled, i.e.,  $\delta_k = 1$  if the *k*-th element of **x**  $\in \mathbb{R}^n$  is sampled (and is therefore known),

otherwise  $\delta_k = 0$ . In the following,  $m \in [0, n]$  and  $m_k \in [0, n]$  $(1 \le k \le n)$  are parameters of the sampling distributions. We consider the following two probability distributions for  $\delta_k$ .

- 1) Uniform Bernoulli Model:  $\mathbf{P}[\delta_k = 1] = m/n$  where  $\mathbf{P}[E]$  denotes that the probability that event *E* occurs. The probability distributions of  $\delta_1, ..., \delta_n$  are assumed to be independent.
- 2) Nonuniform Bernoulli Model:  $\mathbf{P}[\delta_k = 1] = m_k/n$  such that  $\sum_{k=1}^n m_i = nm$ . The probability distributions of  $\delta_1, ..., \delta_n$  are assumed to be independent.

Note that in the above models,  $m, m_1, ..., m_n$  are parameters chosen by the users. It is readily seen from the above definitions that the uniform Bernoulli model is a special case of the nonuniform Bernoulli model, however, we will generally assumed that the  $m_i$ 's in the nonuniform Bernoulli model take on different values. Lastly, note that the number of sampled elements in **x** using the above sampling models can vary from 0 to n, however, the average number of sampled elements is always m for both models.

We assume that the vector **x** is compressible in the transform basis  $\Psi$  where the columns of  $\Psi$  form the basis vectors of the transform basis. Let  $\Omega$  be the set of the indices of the samples, i.e.,  $\Omega = \{k \in [1, n]: \delta_k = 1\}$ . Let also  $\mathbf{I} \in \mathbb{R}^{n \times n}$ denote the identity matrix. We define  $\mathbf{I}_{\Omega}$  be a  $|\Omega|$ -by-*n* matrix such that the *k*-th row of **I** is also a row in  $\mathbf{I}_{\Omega}$  if  $k \in \Omega$ (i.e.,  $\delta_k = 1$ ). With this definition, the vector  $\mathbf{I}_{\Omega}\mathbf{x} \in \mathbb{R}^{|\Omega|}$ contains the sampled elements of **x**.

We will attempt to recover the unknown elements in  $\mathbf{x}$  (i.e., those elements that are not sampled) by using basis pursuit, i.e., by solving the linear programming problem shown in Eq. (2).

In the following we will give results on how the probability of successfully recovering x by using basis pursuit when  $\Omega$  is generated by the uniform and nonuniform Bernoulli models.

## B. Results

Before stating the results, we will need to define a few additional notation. Let  $\theta$  be the coefficients of the vector **x** in the basis  $\Psi$ , i.e.,  $\mathbf{x} = \Psi \theta$ . Since **x** is assumed to be sparse in  $\Psi$ , the vector  $\theta$  is sparse. Let *S* denote the number of nonzero coefficients in  $\theta$ . In addition, we denote the coherence between the identity matrix and the basis  $\Psi$ , by  $\mu(\Psi)$ . Following [3],  $\mu(\Psi)$  is given by  $\sqrt{n} \max_{i,j} \psi_{ij}$  where  $\psi_{ij}$  is the (i, j)-element of  $\Psi$ .

Corollary 1: If the uniform Bernoulli model is used to generate the indices of the samples  $\Omega$  and the basis pursuit method (2) is used to recover the vector **x**, then with a probability exceeding  $1 - \delta$ , it is possible to recover **x** successfully provided that

$$m \ge K\mu^2 \max\left(|S|, \log\left(\frac{3n}{\delta}\right)\right) \log\left(\frac{6n}{\delta}\right)$$
 (3)

 $\square$ 

for some small constant K.

Note that the proof of Corollary 1 can be extracted from the proof in [3]. Although the result in [3] is stated for choosing *m* rows uniformly from I to form the measurement matrix  $I_{\Omega}$  (in other words, **exactly** *m* samples of x are sampled), the proof

relies on using the uniform Bernoulli model as an intermediate step.

For the case of nonuniform Bernoulli model, the probability of recovery is lowered than that of the uniform Bernoulli model. The degradation in recovery probability is stated in the following proposition.

**Proposition 1:** If the uniform Bernoulli model is used to generate the indices of the samples  $\Omega$  and the basis pursuit method (2) is used to recover the vector **x**, then with a probability exceeding  $1 - \kappa \delta$  (where  $\kappa \ge 1$ ), it is possible to recover **x** successfully provided that

$$m \ge K \mu^2 \max\left(|S|, \log\left(\frac{3n}{\delta}\right)\right) \log\left(\frac{6n}{\delta}\right)$$
 (4)

for some small constant K. The value of  $\kappa$  depends on  $m_1$ ,  $m_2$ ,...,  $m_n$  and m, and is given by

$$\kappa = \prod_{k=1}^{n} \kappa_k \tag{5}$$

where

$$\kappa_k = \begin{cases} \frac{m_k}{m} & \text{if } m_k \ge m\\ \frac{n-m_k}{n-m} & \text{if } m_k \le m \end{cases}$$
(6)

## C. Proof of Proposition 1

For a given *n*, there are  $2^n$  different sampling patterns depending on whether each element is sampled or not. We will denote these  $2^n$  sampling patterns by  $\Delta_k$  with  $0 \le k \le 2^n - 1$  where the *k*-th sampling pattern  $\Delta_k$  is  $(\delta_{1k}, ..., \delta_{nk})$  with  $k = \sum_{i=1}^n \delta_{ik} 2^{i-1}$ .

Let  $\Omega_1$  and  $\Omega_2$  denote, respectively, the uniform and nonuniform Bernoulli sampling distributions described in Section IV-A. Let Failure[ $\Omega_1$ ] and Failure[ $\Omega_2$ ] denote, respectively, the event that the uniform Bernoulli and nonuniform Bernoulli sampling method will fail to recover the unknown vector **x**. We can express the probability of these two events as

$$\mathbf{P}[\text{Failure}(\Omega_1)] = \sum_{\substack{k=0\\2^n-1}}^{2^n-1} \mathbf{P}[\text{Failure}(\Delta_k)]\mathbf{P}_{\Omega_1}[\Delta_k] \qquad (7)$$

$$\mathbf{P}[\text{Failure}(\Omega_2)] = \sum_{k=0}^{2^{-1}} \mathbf{P}[\text{Failure}(\Delta_k)] \mathbf{P}_{\Omega_2}[\Delta_k] \qquad (8)$$

where  $\mathbf{P}[\text{Failure}(\Delta_k)]$  is the probability that sampling pattern  $\Delta_k$  does not lead to successfully recovery. In addition,  $\mathbf{P}_{\Omega_1}[\Delta_k]$  and  $\mathbf{P}_{\Omega_2}[\Delta_k]$  are, respectively, that the sampling pattern  $\Delta_k$  is drawn under the uniform Bernoulli model  $\Omega_1$  and the nonunifrom Bernoulli model  $\Omega_2$ .

By using equations (7) and (8), it can be shown that

$$\mathbf{P}[\text{Failure}(\Omega_2)] = \sum_{k=0}^{2^n - 1} \mathbf{P}[\text{Failure}(\Delta_k)] \mathbf{P}_{\Omega_2}[\Delta_k]$$

$$= \sum_{k=0}^{2^n - 1} \mathbf{P}[\text{Failure}(\Delta_k)] \mathbf{P}_{\Omega_1}[\Delta_k] \frac{\mathbf{P}_{\Omega_2}[\Delta_k]}{\mathbf{P}_{\Omega_1}[\Delta_k]}$$

$$\leq \left(\sum_{k=0}^{2^n - 1} \mathbf{P}[\text{Failure}(\Delta_k)] \mathbf{P}_{\Omega_1}[\Delta_k]\right)$$

$$\max_{\Delta_k} \frac{\mathbf{P}_{\Omega_2}[\Delta_k]}{\mathbf{P}_{\Omega_1}[\Delta_k]}$$

$$= \mathbf{P}[\text{Failure}(\Omega_1)] \max_{\Delta_k} \frac{\mathbf{P}_{\Omega_2}[\Delta_k]}{\mathbf{P}_{\Omega_1}[\Delta_k]}$$
(9)

Therefore, the probability of failure using the nonBernoulli model is  $\mathbf{P}_{\Omega_2}[\Delta_k]/\mathbf{P}_{\Omega_1}[\Delta_k]$  (which will be denoted by  $\kappa$ ) times worse than that of the uniform Bernoulli model. It can be shown that  $\kappa$  is equivalently given by

$$\kappa = \max_{\delta_k \in \{0,1\}, k=1, \dots, n} \frac{\prod_{k=1}^n (\delta_k \frac{m_k}{n} + (1 - \delta_k)(1 - \frac{m_k}{n}))}{\prod_{k=1}^n (\delta_k \frac{m}{n} + (1 - \delta_k)(1 - \frac{m_k}{n}))}$$
$$= \prod_{k=1}^n \max_{\delta_k \in \{0,1\}} \frac{\delta_k \frac{m_k}{n} + (1 - \delta_k)(1 - \frac{m_k}{n})}{\delta_k \frac{m}{n} + (1 - \delta_k)(1 - \frac{m_k}{n})}.$$
(10)

It can be shown that the expression on the right-hand-side is maximised if  $\delta_k$  is chosen to be 1 if  $m_k \ge m$ , otherwise  $\delta_k$ should be chosen to be 0. This shows that  $\kappa$  is given by the expression in Proposition 1.

Although Proposition 1 shows that the probability of exact reconstruction for nonuniform compressive sensing is lower than that of uniform sampling, we will show using real WSN data in the next section that the performance degradation is negligible.

# V. NCS APPLICATION: ENERGY NEUTRAL OPERATION FOR RECHARGEABLE WSNS

In this section, we evaluate the performance of the proposed NCS framework in a rechargeable WSN whose nodes have different solar harvest rates. The dominant form of energy consumption in this WSN is sensing. We will demonstrate how NCS can be used to produce accurate temporal-spatial profile for a given energy budget while maintaining energy neutral operation.

## A. Application Description and Experiment Setup

We consider the application of WSN in rainforest monitoring at the Springbrook National Park, which is is part of a World Heritage precinct in Australia. A WSN of 200 nodes is to be deployed at Springbrook by 2011 with the aim to collect microclimate data for improving the understanding of the rainforest restoration processes. The WSN deployment will be completed in three phases, where the phase one deployment has already been completed. Fig. 1 shows the eight nodes deployed in phase one.

Wind speed and wind direction are two important indicators to monitor the rain forest restoration process, therefore each



Fig. 1. Sensor locations of the Springbrook WSN.

of the nodes in the phase one deployment contains these two types of wind sensors. These wind sensors consume substantial amount of energy. In particular, their energy consumption is higher than the radio [19]. Therefore, sensing (or sampling) is the dominant form of energy consumption for each sensor node.

TABLE I Average Solar Energy Harvest Rates of the Nodes

Node ID	Harvest Rates (mA/min)
1	13
2	15
3	13
4	14
5	1
6	16
7	12
8	15

Energy supply is a major design constraint in the Springbrook deployment. In order to cope with the high-energy demand of the wind sensors, each node is equipped with a solar panel to recharge the battery. A key feature of this WSN is that there is a variation in the solar harvesting rate at each node. For nodes out in the open, i.e., not overshadowed by trees, such as node 1–4 and 6–8 in Fig. 1, the solar energy harvesting rate is high. However, for node 5 in Fig. 1, which is situated deep inside the forest, its solar energy harvesting rate is only 5% of the daily solar energy harvest rate of those nodes out in the open. Table I shows the average solar current harvest rates of these eight nodes. Table I also shows that this WSN monitoring application meets Assumption 2 of NCS because of energy supply heterogeneity in the sensor nodes. Intuitively, the node that has a higher energy supply can sample more.

The sampling interval for the wind sensors at the Springbrook site is 5 minutes. We used one month of data which has 8,448 snapshots of both wind speed and wind direction sensor data. Note that a snapshot is the collection of sensor readings at a given time instance. Therefore, a snapshot of wind speed consists of the 8 wind speed readings from the 8 sensors. As in [19], we used a signal of length 1,024, which is made up by "vectorizing" a set of 128 snapshots—this corresponds to choosing N = 8, T = 128 and n = 1,024 in Section III. The 8,448 (= 66 × 128) snapshots are divided into 66 data segments and we apply NCS to each data segment in turn. The presented results are the average over these 66 data segments.



Fig. 2. (a) Compressibility of the DCT coefficients of the wind speed and (b) wind direction signals.

We begin by checking that Assumption 1 (Section II) on the compressibility of the signal holds. We therefore studied the compressibility of the wind sensor data in a number of sparsifying bases including DCT, wavelets (Haar, Symlets) and Fourier. For both wind speed and wind direction, by computing the representation of all the data segments in these sparsifying basis, we observed that out of all the bases, the coefficients in the DCT have the quickest decay. In order to demonstrate the compressibility of the signal, we need to show that the coefficients decay like power law as Eq. (1). In Fig. 2, we plot both the DCT coefficients and power law decay, and it can be seen that the DCT coefficients lie below the power law curve. Therefore, the wind speed and direction signal satisfies Assumption 1, and both assumptions of NCS have been met. Apart from the heterogeneity and sparsity assumptions, to gaurantee an accurate recovery using uniform or nonuniform Bernoulli sampling, the coherence (Section IV-B) of the basis  $\Psi$  should be small. From [3], we know that coherence must be in the range of  $[1, \sqrt{n}]$  and it can be shown that the coherence of the DCT matrix is  $\sqrt{2}$ , which is close the the

lower limit of possible coherence values. Given our sensing data is compressible in the DCT domain, Bernoulli sampling should work well with our data.

# **B.** Distributed Implementation

A centralized approach of energy-aware workload (or sampling rate) allocation typically increases the amount of communications between the nodes and the base station which consumes transmission energy. In this section we present a distributed implementation of NCS, where nodes locally make sampling decision without frequent communication with the base station and thus save the additional energy required by the centralized approach for base to node communication.

Given each data segment consists of data collected from N sensors over T sampling instances, and assume that an application needs to collect on average m samples (out of a total of n = NT samples for each data segment) in order to meet the reconstruction accuracy requirement. In NCS, node j samples at a rate of  $m_j/n$ . This sampling can be conducted locally using the steps as follows:

Step 1: Node j (j = 1, ..., N) generates a random variable  $\delta_j$  using the following distribution function at each sampling time instance:

$$\delta_j = \begin{cases} +1 & \text{with prob. } \frac{m}{T}g_j, \\ 0 & \text{with prob. } 1 - \frac{m}{T}g_j, \end{cases}$$
(11)

where  $g_j$  is the ratio of the energy of a node to the total energy of the nodes, i.e.,  $g_j = E_j / \sum_{j=1}^N E_j$  and  $E_j$  is the energy harvest rate at node j.

Step 2: If  $\delta_j = 1$ , node  $n_j$  samples at that time instance and transmits its measurements to the base station.

The above steps mean that: (1) Node j samples at a rate of  $m_j/n = m/Tg_j$ . (2) The term  $g_j$  is proportional to the energy harvest rate of node j, thus heterogeneity in energy harvest rate will automatically produce nonuniform sampling in the network.

After every T sampling instances, the base station can use the measured samples collected from the sensors to reconstruct the unknown (i.e., not sampled) data in each data segment using the method described in Section IV. Note that the expected number of samples to be collected for each data segments (with n = NT data points) is m.

Note, in our distributed implementation, the sensor nodes need to know the values of m and T. They also need to share the values of their energy harvest rates  $(E_j)$  among them. The communication overhead of NCS is very little because these values do not need to be updated frequently (e.g., once per day). For example, the value of m is set whenever the application reconstruction requirement is known.

## C. Evaluation Results

We now evaluate the performance of NCS framework using the wind speed and wind direction data collected from the Springbrook WSN. We will compare the performance of 4 schemes. The first one is the nonuniform sampling scheme based on *nonuniform Bernoulli* sampling proposed in Section IV with the sampling rate of each node determined by



Fig. 3. Reconstruction accuracy comparison of different schemes with (a) wind speed and (b) wind direction signals.

the method in Section V-B. The second scheme a previously proposed nonuniform sampling method *EAST* [19], which also leverages the energy supply heterogeneity of rechargeable WSNs to support nonuniform sample schedules. However, instead of compressive sensing, EAST is based on a simplified AMS sketching coding/decoding for signal reconstruction. For the reason of completeness, the third scheme to be compared is UEST, which is based on a uniform sample schedule and AMS sketching coding/decoding schemes [22]. The fourth scheme is the *uniform Bernoulli* scheme in Section IV where each node samples with a probability of m/n.

We first compare the reconstruction accuracy of a data segment (i.e.,  $\mathbf{x} \in \mathbb{R}^n$  with n = NT) against the fraction of data points (= m/n) used (It is important to point out that the comparison here does not take into consideration whether the network has sufficient energy to collect the samples given by data fraction m/n. The aim here is to study the reconstruction performance if a given fraction of data is available. The actual amount of data that a network can collect depends on the sampling scheme and will be discussed later in the section). We measure the reconstruction accuracy by using the relative error in reconstructing the vector x. Fig. 3 shows the reconstruction accuracy of different sampling schemes (uniform Bernoulli, nonuniform Bernoulli, UEST and EAST) against different fraction of data points for both wind speed and wind direction data. The figure shows that uniform Bernoulli and nonuniform Bernoulli (both of which are based on compressive sensing) perform significantly better than UEST and EAST (both of which are based on AMS decoder). CS-based methods



Fig. 4. Reconstruction error of (a) wind speed and (b) wind direction signals.

outperform nonCS methods because AMS-decoder needs more data than compressive sensing to achieve the same level of reconstruction accuracy as shown in [22]. Fig. 3 also shows that nonuniform Bernoulli performs very close to uniform Bernoulli, and the difference in relative approximation error is less than 0.2 when a fraction of data points equal to 0.5. This shows that the nonuniform Bernoulli can achieve almost the same accuracy as uniform Bernoulli if the same amount of data is available.

Furthermore, we are interested in the impact of nonuniform sampling on the signal reconstruction accuracy at each node. Fig. 4 shows the mean and standard deviation (among different time snapshots) relative approximation error of all nodes on both wind speed and direction signals using nonuniform Bernoulli sampling with the fraction of data points fixed at 0.3. Fig. 4 shows that node 5 does not have the maximum relative approximation error, though the node has the minimum energy level. Therefore, nonuniform sampling schedule *has no significant impact* on the signal reconstruction accuracy of different nodes.

The above evaluation shows that uniform and nonuniform Bernoulli sampling have comparable reconstruction accuracy but the evaluation does not consider whether the WSN has sufficient energy to sustain the sampling operation. We now impose the additional requirement that the WSN should have energy neutral operation, i.e., the power consumption of each node must be less than the harvest rate of that node. We base our calculations on the solar harvest rate in Table I. We find that uniform Bernoulli sampling can only sustain a sampling rate, expressed as the fraction of data points m/n, of merely 0.1 but nonuniform sampling can achieve 0.5. An explanation of the difference is as follows. First note that node 5 in the network has a low energy harvest rate (see Table I), so node 5 can only use a low sampling rate. Recall that uniform sampling requires that each WSN node uses the same sampling rate and given that node 5 can only use a low sampling rate, this forces all nodes in the network to use a low sampling rate of 0.1 to achieve uniform sampling and energy neutral operation. However, for nonuniform sampling, the network only has to maintain an overall average sampling rate but each node can have a different sampling rate. As a result, nonuniform sampling achieves an overall sampling rate of 0.5, with node 5 sampling at a lower rate while other nodes sampling at a higher rate. This heterogeneous sampling pattern allows an overall higher sampling rate compared with uniform sampling while maintaining energy neutral operation at each node. If we now return to the reconstruction results in Fig. 3 and let us focus on wind speed data. Given that uniform sampling can only achieve a sampling rate of 0.1, its reconstruction error is 0.37. However, nonuniform sampling can realize a rate of 0.5, its reconstruction error is only 0.125. The same discussion also applies to wind direction. Therefore, by exploiting heterogeneity, nonuniform sampling can achieve a better reconstruction accuracy (lower approximation error).

Another evaluation which has been published in [25] is based on the datasets from Intel Berkeley lab which is public on the web. We exploit heterogeneity of the ETXs (the expected number of transmission). The results also support our claim: non-uniform compressive sensing will achieve better reconstruction accuracy with the same energy budget.

## VI. RELATED WORK

There are a number of prior works on investigating how compressive sensing can be used to improve the efficiency of wireless sensor networks, e.g., [1], [5], [14], [18]. The key difference between [1], [5], [14] is how the network acquires compressive sensing projections, which are linear combinations of the sensor readings. This paper [1] suggested to compute these projections by using an additive medium access control channel. While both papers [5], [14] acquire projections by using message passing, this paper [14] acquires projections by constructing an aggregation tree but this paper [5] uses adaptively compressive sensing to choose the projection coefficients. A common theme of [1], [5], [14] is to efficiently acquire projections in WSNs. This paper [18] has a different emphasis from the other three in that if focused on finding a good basis, i.e., it studies the choice of basis matrix  $\Psi$  (see Section III). Our work in this paper differs from these four papers in two aspects. Firstly, all these four papers assume that sensors sample the physical phenomenon at each sampling instance but we consider nonuniform sampling in this paper, which means that some sensors do not sample at certain sampling instances. Secondly, these four papers assume that energy consumption in the WSNs is dominated by wireless transmissions, however, we consider different case where energy consumption can be dominated by sensing.

The CS based methods discussed earlier use a dense projection matrix, which requires all the data points a signal vector to be collected. However, since only a few projections need to be transmitted, the proposed methods could save the transmission energy. Our CS implementation is different from the earlier methods since we use a sparse projection matrix and thus do not require to collect all the data points of the signal vector. A CS based data gathering approach is presented in [17] which investigates the impact of a routing topology generated sparse projection matrix on the accuracy of the approximation. Our work is different from theirs since our projection matrix is not based on the routing topology rather it is populated based on the energy profile of the sensors.

Different techniques apart from CS have been used in the past to enable adaptive sesning exploiting the temporal, spatial or spatial-temporal [2], [6], [8], [13], [24] correlation of the signal. Though both our sensing/transmission approaches exploit the temporal-spatial correlation of the data points, we have considered nonuniform energy profile of the sensors, which is different from the existing literature.

Work presented in [12] and [16] propose harvest-aware sampling approaches, where sensors are assigned sampling workload based on their harvested energy level. However, the focus of these papers are not on signal approximation from the network.

Work presented in [19] uses nonuniform sampling to improve the efficiency of solar powered WSNs whose energy consumption is dominated by sensing. However, unlike CS, the algorithm proposed in [19] named EAST, is based on a sparse approximation method called "sparse random projections" proposed in [22]. In Section IV compare the performance of EAST against the algorithm proposed in this paper and we find that our proposed algorithm in this paper outperforms EAST.

Communication overhead heterogeneity has also been exploited to extend sensor network lifetime in [11] and [10]. However, [11] is designed to satisfy an application's acceptable tolerance of aggregation queries (such as min, max, sum, mean) with imprecise and inaccurate samples. On the other hand, NCS is designed to recover the whole signals with some approximation errors. The authors assume there are heterogenous sensor nodes, with heterogeneous radios and initial energy supplies, in the networks [10]. NCS can work with sensor networks that have homogenous radios and initial energy supplies.

#### VII. CONCLUSION

In this paper, we consider a large scale wireless sensor network (WSN) measuring spatio-temporal correlated physical phenomena, i.e., compressible signals. At the same time, we also consider that there is there is heterogeneity in resource supply in the WSNs, which is a common phenomenon. We proposed a a framework to address the problem of heterogeneous sensor sample schedule (nonuniform sampling) signal reconstruction by extending Compressive Sensing (CS) theory, to exploit the heterogeneity in a WSN that monitor compressible signals, in order to improve network performance. We prove that, similar to its uniform sampling counterparts, nonuniform CS can also recover compressible signal accurately with a high probability. We evaluated proposed NCS with a real WSN application, which has resource supply heterogeneity. We presented a distributed implementation of NCS framework that introduced very little communication overheads, and show that, compared to previously proposed approaches based on traditional CS and sparse approximation respectively, NCS achieved similar signal approximation accuracy with significantly less samples (energy consumption).

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